

Mathematical Analysis

This discipline combines many of the trigonometric, geometric, and algebraic techniques needed to prepare students for the study of calculus and strengthens their conceptual understanding of problems and mathematical reasoning in solving problems. These standards take a functional point of view toward those topics. The most significant new concept is that of limits. Mathematical analysis is often combined with a course in trigonometry or perhaps with one in linear algebra to make a yearlong precalculus course.

Students:

- 1.0 Are familiar with, and can apply, polar coordinates and vectors in the plane. In particular, they can translate between polar and rectangular coordinates and can interpret polar coordinates and vectors graphically.
- 2.0 Are adept at the arithmetic of complex numbers. They can use the trigonometric form of complex numbers and understand that a function of a complex variable can be viewed as a function of two real variables. They know the proof of DeMoivre’s theorem.
- 3.0 Can give proofs of various formulas by using the technique of mathematical induction.
- 4.0 Know the statement of, and can apply, the fundamental theorem of algebra.
- 5.0 Are familiar with conic sections, both analytically and geometrically:
  - 5.1 Students can take a quadratic equation in two variables; put it in standard form by completing the square and using rotations and translations, if necessary; determine what type of conic section the equation represents; and determine its geometric components (foci, asymptotes, and so forth).

- 5.2 Can take a geometric description of a conic section—for example, the locus of points whose sum of its distances from (1, 0) and (-1, 0) is 6—and derive a quadratic equation representing it.
- 6.0 Find the roots and poles of a rational function and can graph the function and locate its asymptotes.
- 7.0 Demonstrate an understanding of functions and equations defined parametrically and can graph them.
- 8.0 Are familiar with the notion of the limit of a sequence and the limit of a function as the independent variable approaches a number or infinity. They determine whether certain sequences converge or diverge.

Senate Bill 2X  
High School Exit Exam Highlights

- Senate Bill 2X requires all students completing grade twelve to pass a high school exit exam in language arts and math commencing in 2003–04.
- The bill requires the State Superintendent of Public Instruction to develop and the State Board of Education to approve the exam by October 1, 2000.
- Beginning in 2000–01, grade nine students will be eligible to take the exam.
- Beginning in 2001–02, grade ten students will be required to take the exam.
- The law does not make the exam a requirement for graduation until 2003–04.
- If a pupil does not possess sufficient English language skills to be assessed by the exit exam, the district may defer the requirement that the student pass the exam “for a period of up to 24 calendar months of enrollment in the California public school system until the pupil has completed six months of instruction in reading, writing, and comprehension in the English language.”

College Entrance Requirements

Parents generally know that many colleges require good high school grades for admission. Although grades are important, students do not have to have top grades to get into college. There are colleges for every student. You should also know that students need to take a specific series of college preparatory classes in high school, and the minimum requirements vary depending on the selected college or university. The a–g requirements noted below are submitted by the Regents of the University of California and are generally the most rigorous:

- a. An English class every semester of every year for four years.
- b. A mathematics class every semester of every year for three years, including algebra and geometry. Four years are recommended.
- c. Two years of a laboratory science beyond the ninth grade. An additional year is recommended.
- d. Two years of history–social science, which are to include U.S. government, world history, culture, and geography.
- e. Two years of the same language other than English.
- f. Two years of college preparatory electives in addition to those required in “a–e” above.
- g. One year of visual and performing arts, effective for the entering class of 2003.

Every high school has a list of acceptable classes and can tell you how many should be taken. At least one class in the area of visual or performing arts is a good choice for many students.

To gain admission to college, your children must also take either the Scholastic Assessment Test (SAT) or the American College Test (ACT) and submit the scores. Find out when the tests are given and be sure your children sign up to take one of them.

ALGEBRA II

The  
California  
Mathematics  
Content  
Standards

CALIFORNIA  
DEPARTMENT  
OF EDUCATION

2001

# The California Mathematics Content Standards

ACADEMIC CONTENT STANDARDS IN CRITICAL curriculum areas are an important part of educational reform in California. This brochure provides an overview of the California Department of Education’s approach to meeting the academic needs of your child in the core curricular areas (math, science, history-social science, and language arts). A copy of the mathematics content standards adopted by the State Board of Education is included.

Well-communicated standards spell out what students learn in a specific subject. School districts must adopt the state standards or use them as a foundation for creating their own district standards. When a school district develops standards, they must be as rigorous and challenging as the state standards.



The standards for grades eight through twelve are organized differently from those for kindergarten through grade seven. In this section strands are not used for organizational purposes as they are in the elementary grades because the mathematics studied in grades eight through twelve falls naturally under discipline headings: algebra, geometry, and so forth. Many schools teach this material in traditional courses; others teach it in an integrated fashion.

To allow local educational agencies and teachers flexibility in teaching the material, the standards for grades eight through twelve do not mandate that a particular discipline be initiated and completed in a single grade. The core content of these subjects must be covered; students are expected to achieve the standards however these subjects are sequenced.

Standards are provided for algebra I, geometry, algebra II, trigonometry, mathematical analysis,

linear algebra, probability and statistics, Advanced Placement probability and statistics, and calculus. Many of the more advanced subjects are not taught in every middle school or high school. Moreover, schools and districts have different ways of combining the subject matter in these various disciplines. For example, many schools combine some trigonometry, mathematical analysis, and linear algebra to form a precalculus course. Some districts prefer offering trigonometry content with algebra II.

What is described in this section are standards for the academic content by discipline; the document does not endorse a particular choice of structure for courses or a particular method of teaching the mathematical content.

## Algebra II

This discipline complements and expands the mathematical content and concepts of algebra I and geometry. Students who master algebra II will gain experience with algebraic solutions of problems in various content areas, including the solution of systems of quadratic equations, logarithmic and exponential functions, the binomial theorem, and the complex number system.

### Students:

- 1.0** Solve equations and inequalities involving absolute value.
- 2.0** Solve systems of linear equations and inequalities (in two or three variables) by substitution, with graphs, or with matrices.
- 3.0** Are adept at operations on polynomials, including long division.
- 4.0** Factor polynomials representing the difference of squares, perfect square trinomials, and the sum and difference of two cubes.
- 5.0** Demonstrate knowledge of how real and complex numbers are related both arithmetically and graphically. In particular, they can plot complex numbers as points in the plane.

- 6.0** Add, subtract, multiply, and divide complex numbers.
- 7.0** Add, subtract, multiply, divide, reduce, and evaluate rational expressions with monomial and polynomial denominators and simplify complicated rational expressions, including those with negative exponents in the denominator.
- 8.0** Solve and graph quadratic equations by factoring, completing the square, or using the quadratic formula. Students apply these techniques in solving word problems. They also solve quadratic equations in the complex number system.
- 9.0** Demonstrate and explain the effect that changing a coefficient has on the graph of quadratic functions; that is, students can determine how the graph of a parabola changes as *a*, *b*, and *c* vary in the equation  $y = a(x-b)^2 + c$ .
- 10.0** Graph quadratic functions and determine the maxima, minima, and zeros of the function.
- 11.0** Prove simple laws of logarithms.
  - 11.1** Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.
  - 11.2** Judge the validity of an argument according to whether the properties of real numbers, exponents, and logarithms have been applied correctly at each step.
- 12.0** Know the laws of fractional exponents, understand exponential functions, and use these functions in problems involving exponential growth and decay.
- 13.0** Use the definition of logarithms to translate between logarithms in any base.
- 14.0** Understand and use the properties of logarithms to simplify logarithmic numeric expressions and to identify their approximate values.

- 15.0** Determine whether a specific algebraic statement involving rational expressions, radical expressions, or logarithmic or exponential functions is sometimes true, always true, or never true.
- 16.0** Demonstrate and explain how the geometry of the graph of a conic section (e.g., asymptotes, foci, eccentricity) depends on the coefficients of the quadratic equation representing it.
- 17.0** Given a quadratic equation of the form  $ax^2 + by^2 + cx + dy + e = 0$ , students can use the method for completing the square to put the equation into standard form and can recognize whether the graph of the equation is a circle, ellipse, parabola, or hyperbola. Students can then graph the equation.
- 18.0** Use fundamental counting principles to compute combinations and permutations.
- 19.0** Use combinations and permutations to compute probabilities.
- 20.0** Know the binomial theorem and use it to expand binomial expressions that are raised to positive integer powers.
- 21.0** Apply the method of mathematical induction to prove general statements about the positive integers.
- 22.0** Find the general term and the sums of arithmetic series and of both finite and infinite geometric series.
- 23.0** Derive the summation formulas for arithmetic series and for both finite and infinite geometric series.
- 24.0** Solve problems involving functional concepts, such as composition, defining the inverse function and performing arithmetic operations on functions.
- 25.0** Use properties from number systems to justify steps in combining and simplifying functions.